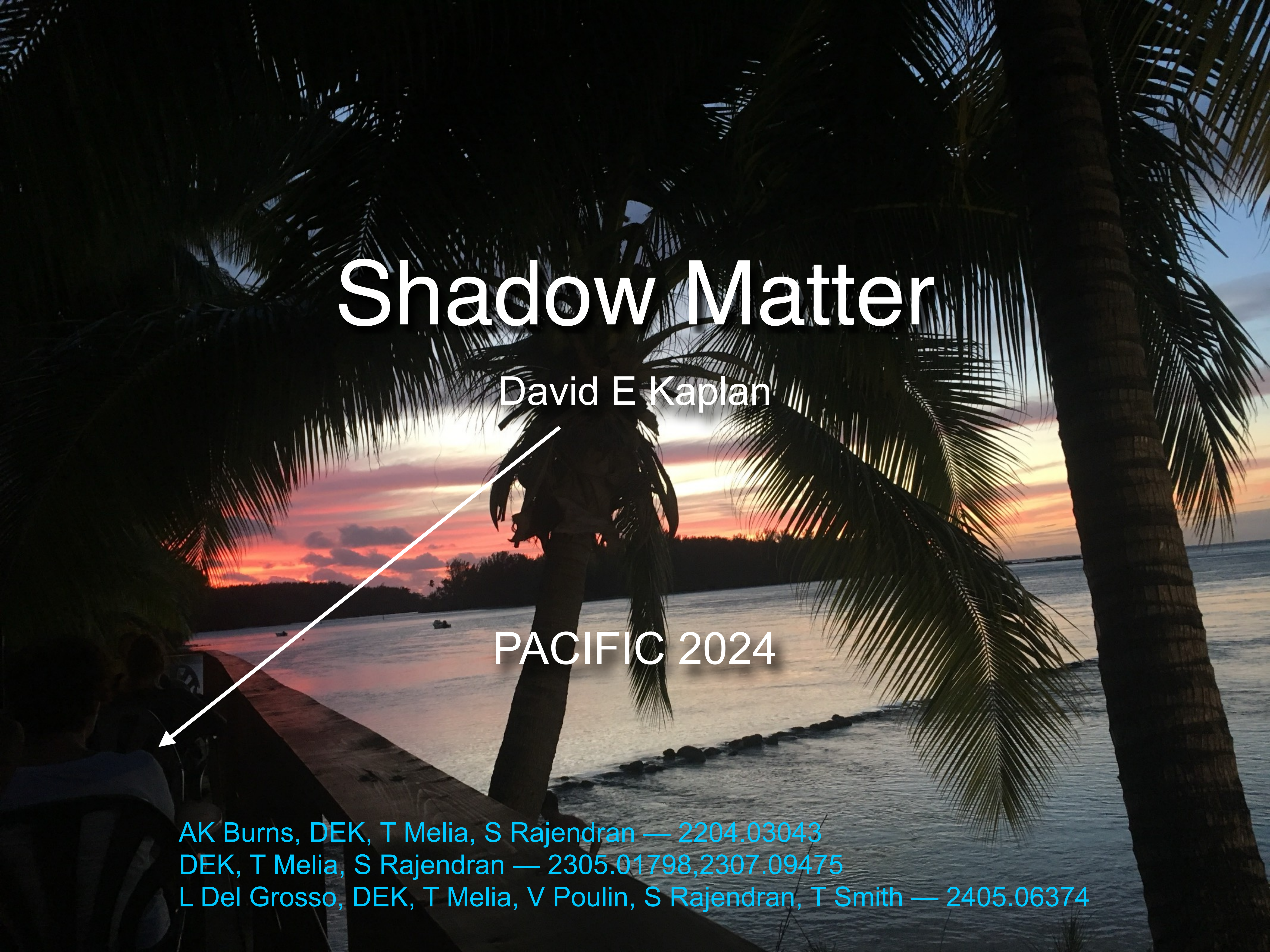

Shadow Matter

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Gauge Theories and Gravity without Constraints

The QFT of the world produces the classical theories that we discovered first (GR, EM, Newton’s laws)

We explore EM and GR without standard constraints on the initial states and see some new effects

Classical Equations of Motion

$$S = \int dt L(q, \dot{q})$$

$$\frac{\delta S}{\delta q} = 0$$

Hamiltonian can be build from: $p \equiv \frac{\delta L}{\delta \dot{q}}$

Ehrenfest

$$\left[\hat{q}_i , \hat{p}^j\right] = i\delta_i^j$$

for $H = \dfrac{p^2}{2m} + V(q)$

$$\partial_t q_i = i\left[H, q_i\right] = \frac{\partial H}{\partial p^i} = \frac{p_i}{m}$$

$$\partial_t p^i = i\left[H, p^i\right] = -\frac{\partial H}{\partial q_i} = -\frac{\partial V(q)}{\partial q_i}$$

(up to commutators)

Note: If there is a q_k without a p_k, we lose an e.o.m.

EM

EM

classical: $$S = \int d^4x \mathscr{L} = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A_\mu J^\mu + \cdots \right]$$

Gauss' Law $$\frac{\delta S}{\delta A_0} = \underbrace{\partial_\mu F^{\mu 0}}_{\nabla \cdot E} - J^0 = 0$$

Ampere's Law $$\frac{\delta S}{\delta A_i} = \underbrace{\partial_\mu F^{\mu i}}_{\dot{E} + \nabla \times B} - J^i = 0$$

conjugate momenta:

$$\pi^i \equiv \frac{\delta \mathscr{L}}{\delta \dot{A}_i} = \partial^0 A^i - \partial^i A^0 \equiv -E^i \qquad \pi^0 \equiv \frac{\delta \mathscr{L}}{\delta \dot{A}_0} = 0$$

EM

Hamiltonian: $$H = \int d^3x \mathscr{H}$$

$$\mathscr{H} = \frac{1}{2}\left(\mathbf{E}^2 + \mathbf{B}\right)^2 - A_0(\nabla \cdot \mathbf{E} - J^0) + \mathbf{A} \cdot \mathbf{J} + \cdots$$

$$B = \nabla \times \mathbf{A}$$

Choose A_0 — gauge freedom makes it plausible that choice doesn't affect dynamics.

Simple choice: $A_0 = 0$ (Weyl gauge)

Invent quantum version

Commutators canonical: $$\left[E^i(\mathbf{x}), A_j(\mathbf{y})\right] = i\,\delta^i_j\,\delta(\mathbf{x} - \mathbf{y})$$

EM

From the Schrödinger Eq:

$$\partial_t \langle \mathbf{A} \rangle = i \langle \left[H, \mathbf{A} \right] \rangle = \langle \mathbf{E} \rangle$$

$$\partial_t \langle \mathbf{E} \rangle = i \langle \left[H, \mathbf{E} \right] \rangle = \langle \nabla \times \mathbf{B} + \mathbf{J} \rangle$$ Ampere's Law

Gauss' Law? $G \equiv \nabla \cdot \mathbf{E} - J^0$

Note: $\left[H, G \right] = 0$ (G's conjugate doesn't appear!)

Can require: $G | \psi \rangle_{\text{phys}} = 0$

EM

Ampere's Law $$\partial_t\langle \mathbf{E}\rangle = i\langle\left[H,\mathbf{E}\right]\rangle = \langle\,\nabla\times\mathbf{B}+\mathbf{J}\rangle$$

Gauss' Law — Can require:

$$G \equiv \nabla\cdot\mathbf{E} - J^0$$

$$G\,|\,\psi\rangle_{\rm phys} = 0$$

Could instead require: $$G\,|\,\psi\rangle_{\rm phys} = \rho_{\rm sh}(\mathbf{x})\,|\,\psi\rangle_{\rm phys}$$

Or even: $$\langle\psi\,|\,G\,|\,\psi\rangle = 0 \;\text{ or }\; \rho_{\rm sh}(\mathbf{x})$$

These appears as normal QED, potentially with a static charge density b.g.

$$\langle\,\nabla\cdot\mathbf{E} - J^0 - \rho_{\rm sh}(\mathbf{x})\rangle = 0$$

equivalent physics to infinite mass charge distribution

Gravity (toy)

Gravity: minisuperspace

zero-mode only (FRW): $$ds^2 = -N(t)^2 dt^2 + a(t)^2 d\mathbf{x}^2$$

classical: $$S = \int d^4x \sqrt{-g}\,\Big(M_{\rm pl}^2 R + \underbrace{\mathscr{L}_{\rm matter}(\phi)}_{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \cdots} \Big) + S_{GHY}$$

$$\frac{\delta S}{\delta N} = a^3 \left(6M_{\rm pl}^2 \frac{\dot a^2}{N^2 a^2} - \frac{\dot\phi^2}{2N^2} - V(\phi) \right) = 0$$ 1st Friedmann Eq

$$\frac{\delta S}{\delta a} = 3Na^2 \left(4M_{\rm pl}^2 \left(\frac{\ddot a}{N^2 a} + \frac{\dot a^2}{2N^2 a^2} - \frac{\dot a \dot N}{N^3 a} \right) + \frac{\dot\phi^2}{2N^2} - V(\phi) \right) = 0$$ 2nd Friedmann Eq

$$\frac{\delta S}{\delta \phi} = -\frac{a^3}{N} \left(\ddot\phi + 3\frac{\dot a}{a}\dot\phi - \frac{\dot N \dot\phi}{N} + N^2 \frac{\partial V(\phi)}{\partial\phi} \right)$$ matter EOM

Gravity: minisuperspace

zero-mode only (FRW): $ds^2 = -N(t)^2 dt^2 + a(t)^2 d\mathbf{x}^2$

classical: $S = \int d^4x \sqrt{-g}\,\left(M_{\rm pl}^2 R + \underbrace{\mathscr{L}_{\rm matter}(\phi)}_{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \cdots} \right) + S_{GHY}$

$$\left.\frac{\delta S}{\delta N}\right|_{N=1} = a^3 \left(6M_{\rm pl}^2 \frac{\dot{a}^2}{a^2} - \frac{1}{2}\dot{\phi}^2 - V(\phi) \right) = 0$$

1st Friedmann Eq

$$\left.\frac{\delta S}{\delta a}\right|_{N=1} = 3a^2 \left[4M_{\rm pl}^2 \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{2a^2} \right) + \frac{1}{2}\dot{\phi}^2 - V(\phi) \right] = 0$$

2nd Friedmann Eq

$$\left.\frac{\delta S}{\delta \phi}\right|_{N=1} = -a^3 \left(\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} \right)$$

matter EOM

Gravity: minisuperspace

conjugate momenta:

$$\pi_a \equiv \frac{\delta \mathscr{L}}{\delta \dot{a}} = -6M_{\rm pl}^2 \frac{a\dot{a}}{N} \qquad \pi_N \equiv \frac{\delta \mathscr{L}}{\delta \dot{N}} = 0 \qquad \pi_\phi = \frac{\delta \mathscr{L}}{\delta \dot{\phi}} = \frac{a^3}{N}\dot{\phi}$$

Hamiltonian:

$$H = \left[\pi \dot{a} + \pi_\phi \dot{\phi} - \mathscr{L}\right]_{\dot{a}=\cdots,\dot{\phi}=\cdots} = -\frac{N}{24M_{pl}^2 a}\pi^2 + \frac{N}{2a^3}\pi_\phi^2 + Na^3 V(\phi)$$

$$= N\tilde{H}(\pi_a, a, \pi_\phi, \phi)$$

Gravity: minisuperspace

Schrödinger eq:

$$i\frac{d}{dt}|\psi\rangle = N(t)\,\tilde{H}\,|\psi\rangle \quad \rightarrow i\frac{d}{N(t)dt}|\psi\rangle = \tilde{H}\,|\psi\rangle$$

N maintains time reparameterization invariance.

Simple choice: $N = 1$

$$\partial_t\langle a\rangle = i\langle[H,a]\rangle \rightarrow \langle\pi_a\rangle = \langle -6M_{\rm pl}a\dot{a}\rangle$$

(choosing operator ordering wisely)

$$\partial_t\langle\pi_a\rangle = i\langle\left[H,\pi_a\right]\rangle \rightarrow$$

(replacing π_a's with $\dot{a}$'s, assuming classical states)

$$\longrightarrow \frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{M_{\rm pl}^2}p$$

2nd Friedmann Eq

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Gravity: minisuperspace

1st Friedmann Eq? From classical:

$$0 = \frac{\delta S}{\delta N} = \frac{\delta \int \pi_a \dot{a} + \pi_\phi \dot{\phi} - N\tilde{H}}{\delta N} \longrightarrow \tilde{H} = 0$$

The 1st Friedmann Eq is the *Hamiltonian Constraint*

Standard treatment (like Gauss): $\tilde{H}\,|\,\psi\rangle_{\rm phys} = 0$ (Wheeler-deWitt)

??? No Schrödinger equation ???

The "problem of time" in quantum gravity

Simple fix: $\langle\psi\,|\,H\,|\,\psi\rangle = 0$

Gravity: minisuperspace

1st Friedmann Eq? From classical:

$$0 = \frac{\delta S}{\delta N} = \frac{\delta \int \pi_a \dot{a} + \pi_\phi \dot{\phi} - N\tilde{H}}{\delta N} \longrightarrow \tilde{H} = 0$$

Simple fix: $\langle \psi | \tilde{H} | \psi \rangle = 0$

For classical states: $\tilde{H} = a^3 \left(3M_{\rm pl}^2 \left(\frac{\dot{a}}{a} \right)^2 - \rho \right) = 0$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Gravity: minisuperspace

Could choose: $\langle\psi|\tilde{H}|\psi\rangle = \mathbb{C}$

$\mathbb{C}$ is constant as $\left[\tilde{H},\tilde{H}\right] = 0$

For classical states: $\tilde{H} = a^3\left(3M_{\rm pl}^2\left(\frac{\dot{a}}{a}\right)^2 - \rho\right) = \mathbb{C}$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\rm pl}^2}\left(\rho + \frac{\mathbb{C}}{a^3}\right)$$

behaves as another matter component

GR

GR

$$S = \int d^4x \, \underbrace{(\sqrt{-g}\, M_{\rm pl}^2 R \; + \sqrt{-g}\, \mathscr{L}_{\rm matter}(\phi))}_{\mathscr{L}_g} \; + \; S_{GHY}$$

classical:

$$\frac{\delta S}{\delta g_{\mu\nu}} = \sqrt{-g}\, \left(M_{\rm pl}^2 G^{\mu\nu} - T^{\mu\nu} \right) = 0$$

conjugates:

$$\frac{\delta \mathscr{L}_g}{\delta \dot{g}_{ij}} \equiv \pi^{ij} \qquad\qquad \frac{\delta \mathscr{L}_g}{\delta \dot{g}_{0\mu}} = 0$$

construct Hamiltonian in easy gauge: $g_{0\mu} = -\,\delta_{0\mu}$ $\gamma_{ij} \equiv g_{ij}$

(synchronous)

$$\left[\gamma_{ij}(\mathbf{x}), \pi^{kl}(\mathbf{y})\right] = i\, \delta^k_{(i}\delta^l_{j)}\, \delta(\mathbf{x}-\mathbf{y})$$

$$\mathscr{H} = \frac{1}{\sqrt{\gamma}}(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2) - \sqrt{\gamma}\; {}^{(3)}R + \cdots$$

GR

EOM: $$\partial_t \langle \pi^{ij} \rangle \sim -\left\langle \frac{\delta H}{\delta \gamma_{ij}} \right\rangle \longrightarrow G^{ij} = 8\pi G T^{ij} \qquad \left(\frac{\delta S}{\delta g_{ij}} = 0 \right)$$

(under rug: coherent states, operator ordering)

Remaining equations constraints — impose on the initial states

$$\langle \mathscr{H} \rangle = 0$$ Hamiltonian $$\longrightarrow M_{\mathrm{pl}}^{-2}\sqrt{-g}\, G^{00} - \sqrt{-g}\, T^{00} = 0$$

$$\langle \chi^i \rangle = 0$$ Momentum $$\longrightarrow M_{\mathrm{pl}}^{-2}\sqrt{-g}\, G^{0i} - \sqrt{-g}\, T^{0i} = 0$$

$$\langle \mathscr{H} \rangle = \mathbb{H} \longrightarrow M_{\mathrm{pl}}^{-2}\sqrt{-g}\, G^{00} - \sqrt{-g}\, T^{00} = \mathbb{H}$$

$$\langle \chi^i \rangle = \mathbb{P}^i \longrightarrow M_{\mathrm{pl}}^{-2}\sqrt{-g}\, G^{0i} - \sqrt{-g}\, T^{0i} = \mathbb{P}^i$$

GR

$$G^{00} = 8\pi G\left(T^{00} + \frac{\mathbb{H}}{\sqrt{-g}}\right)$$

$$G^{0i} = 8\pi G\left(T^{0i} + \frac{\mathbb{P}^i}{\sqrt{-g}}\right)$$

$$G^{ij} = 8\pi G T^{ij}$$

$\longrightarrow$

$$G^{\mu\nu} = 8\pi G(T^{\mu\nu} + T^{\mu\nu}_{\text{sh}})$$

$$T^{\mu\nu}_{\text{sh}} \equiv \frac{1}{\sqrt{-g}}\begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

$\mathbb{H}, \mathbb{P}^i$ are constrained functions

(identity) $\nabla_\mu G^{\mu\nu} = 0$

(EOM) $\nabla_\mu T^{\mu\nu} = 0$

$$\longrightarrow \nabla_\mu T^{\mu\nu}_{\text{sh}} = 0 \longrightarrow$$

$$\partial_0 \mathbb{H} + \partial_i \mathbb{P}^i = 0$$

$$\partial_0(g_{ij}\mathbb{P}^j) = 0$$

GR

limit: $\mathbb{H}(\mathbf{x}) = \overline{\mathbb{H}} + \delta\mathbb{H}(\mathbf{x})$ $\mathbb{P}^i \equiv \delta\mathbb{P}^i$

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j$$

Perturbative expansion around homogeneity

constraints: $\partial_0\mathbb{H} + \partial_i\mathbb{P}^i = 0$, $\partial_0(g_{ij}\mathbb{P}^j) = 0$ at linear order: $\partial_0(a^2\,\delta\mathbb{P}^j) = 0$

$\delta\mathbb{P}^i \sim a^{-2}$ and $\delta\mathbb{P}^i/\sqrt{-g} \sim a^{-5}$

Redshift quickly away outside horizon

GR

limit: $\mathbb{P}^i = 0$

if $\mathbb{H}(\mathbf{x}) > 0$ everywhere, then can do a spatial coordinate redefinition:

$$\frac{\mathbb{H}(\mathbf{x})}{\sqrt{-g}} \longrightarrow \frac{\overline{\mathbb{H}}}{\sqrt{-g}}$$

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j \qquad h = h_{ij}\delta^{ij}$$

at linear order $$T^{00}_{\rm sh} = (\bar{\rho}_{\rm sh} + \delta\rho_{\rm sh}) = \frac{\overline{\mathbb{H}}}{\sqrt{-g}} \simeq \frac{\overline{\mathbb{H}}}{a^3}(1 - h/2)$$

or $\dot{\delta} = -\dot{h}/2$

Evolves as pressure-less dust

GR + EM

covariant derivatives

$$\nabla_\mu F^{\mu\nu} = (J^\nu + J^\nu_{\mathrm{sh}}) \qquad \nabla_\mu J^\mu_{\mathrm{sh}} = 0$$

$$\sqrt{-g}\, J^\mu_{\mathrm{sh}} \equiv \{\mathbb{J}(\mathbf{x}), 0, 0, 0\}$$

↑ time independent

$$J^\mu_{\mathrm{sh}} = \rho^{ch}_{\mathrm{sh}} v^\mu \qquad v^\mu = \{1, 0, 0, 0\}$$

synchronous gauge

shadow charge density follows geodesics and does not respond to electromagnetic fields directly

Shadow Matter

Loosening the initial conditions of GR allows for source terms that could explain why we think we see dark matter

New source terms for EM produce a charged component of the fake dark matter. Could effect the CMB, BBN, galactic dynamics, and direct detection.

New source terms could violate NEC with no microscopic instabilities. New phenomena possible

If Shadow Matter is most or all of dark matter, it is in conflict with inflation. Worth exploring new ideas for initial conditions.


Thank you!